$f(x) = x^3 + ax^2 + bx + c$

a) per quali valori di a, b, c si ha $\underbrace{f'(1) = f'(-1) = 0}_{1}$ e $\underbrace{\int_{-1}^{1} f(x)\,dx = 4}_{2}$?

b) trovati a, b, c, calcola $\lim_{x \to +\infty} \frac{\int_0^x f(t)\,dt}{x^4}$

$\rightarrow$ calcolo $f'(x)$: $f'(x) = 3x^2 + 2ax + b$

impongo le condizioni richieste:

1 $\begin{cases} 3(1)^2 + 2a(1) + b = 0 \\ 3(-1)^2 + 2a(-1) + b = 0 \end{cases} \Rightarrow \begin{cases} 3 + 2a + b = 0 \\ 3 - 2a + b = 0 \end{cases} \Rightarrow \begin{cases} 4a = 0 \\ \end{cases} \Rightarrow \begin{cases} a = 0 \\ 3 + b = 0 \end{cases} \Rightarrow \boxed{\begin{cases} a = 0 \\ b = -3 \end{cases}}$

$\Rightarrow f(x) = x^3 - 3x + c$

2 $\int_{-1}^{1} f(x)\,dx = 4 \Rightarrow \int_{-1}^{1} x^3 - 3x + c\,dx = \left[\frac{x^4}{4} - \frac{3x^2}{2} + cx\right]_{-1}^{1} = \left[\frac{1}{4} - \frac{3}{2} + c - \frac{1}{4} + \frac{3}{2} + c\right] = 4$

$\Rightarrow 2c = 4 \Rightarrow \boxed{c = 2}$

$\Rightarrow$ risp. a) $\boxed{f(x) = x^3 - 3x + 2}$

ora: $\lim_{x \to +\infty} \frac{\int_0^x t^3 - 3t + 2\,dt}{x^4} = \lim_{x \to +\infty} \frac{\left[\frac{t^4}{4} - \frac{3t^2}{2} + 2t\right]_0^x}{x^4} = \lim_{x \to +\infty} \left(\frac{x^4}{4} - \frac{3x^2}{2} + 2x\right) \cdot \frac{1}{x^4} =$

$\lim_{x \to +\infty} \frac{1}{4} - \frac{3}{2x^2} + \frac{2}{x^3} = \frac{1}{4}$

$\Rightarrow$ risp b) $\boxed{\frac{1}{4}}$