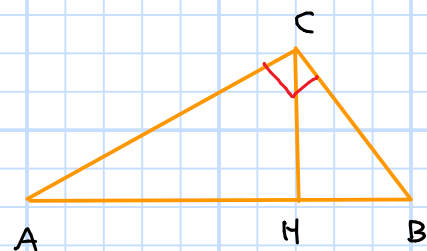


$$\overline{OR} = \frac{r}{3}$$

$$\overline{PB} = ?$$

$$\text{Pongo } \overline{PB} = x$$

In generale.



In un triangolo rettangolo l'altezza relativa all'ipotenusa "crea" due triangoli simili

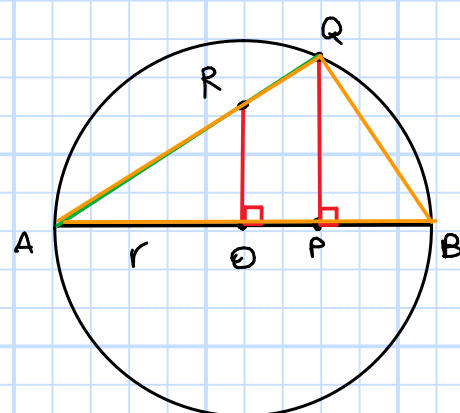
$\triangle AHC \sim \triangle HBC$  perché hanno tutti gli angoli congruenti.

$$\Rightarrow \overline{AH} : \overline{HC} = \overline{HC} : \overline{HB}$$

Considerazione

①

$$\boxed{\overline{PB} = x}$$



$$\textcircled{2} \quad \overline{AP} : \overline{PQ} = \overline{AO} : \overline{OR} \quad (\text{Triangoli simili})$$

$$\Rightarrow (2r - x) : \overline{PQ} = r : \frac{r}{3} \Rightarrow$$

$$\Rightarrow \frac{2r - x}{\overline{PQ}} = r \cdot \frac{3}{r} \Rightarrow \boxed{\frac{2r - x}{3} = \overline{PQ}}$$

Per la considerazione ①  $\overline{AP} : \overline{PQ} = \overline{PQ} : \overline{PB}$  (il triangolo  $\triangle ABQ$  è rettangolo)

$\Rightarrow$  per la ②  $\Rightarrow \overline{AO} : \overline{OR} = \overline{PQ} : \overline{PB}$  (dato che  $\overline{AP} : \overline{PQ} = \overline{AO} : \overline{OR}$ )  $\Rightarrow$

$$r : \frac{r}{3} = \frac{2r - x}{3} : x \Rightarrow$$

$$3 = \frac{2r - x}{3} \cdot \frac{1}{x} \Rightarrow 9 = \frac{2r - x}{x} \Rightarrow 9x = 2r - x \Rightarrow$$

$$10x = 2r \Rightarrow x = \frac{2r}{10} \Rightarrow \boxed{x = \frac{r}{5}}$$