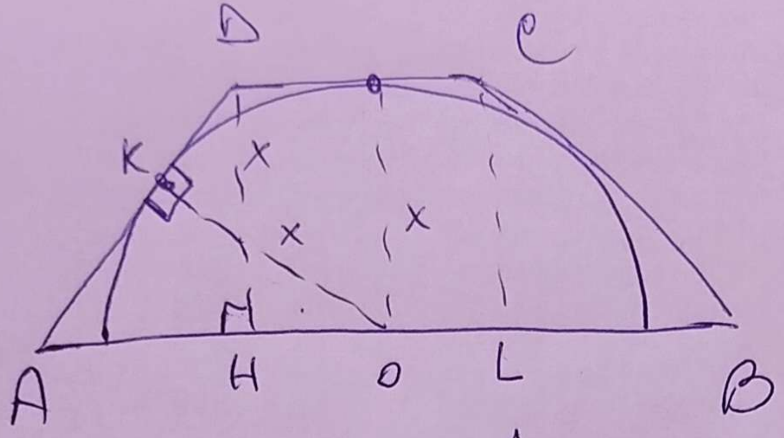


$\overline{AB} = 4$

$AD \cong BC$

$\overline{OK} = \overline{DH} = x$

Confronto i triangoli
AOK e DAH:

$A\hat{K}O \cong A\hat{H}D = 90°$

$K\hat{A}O \equiv D\hat{A}H$ angolo in comune

$OK \cong DH$ raggi della stessa semicirconferenza

2° criterio di congruenza dei triangoli $\Rightarrow AOK \cong DAH$,

in particolare:

$AO = AD$ e $AK = AH$

$AO = \frac{AB}{2} = 2 \Rightarrow AD = AO = 2$

Nel triang rettangolo $A\hat{O}K$ è

$\overline{AK} = \sqrt{\overline{AO}^2 - \overline{OK}^2} = \sqrt{4 - x^2}$

$\Rightarrow \overline{AH} = \overline{AK} = \sqrt{4 - x^2}$

$\overline{DC} = \overline{HL} = 2 \cdot \overline{OH} = 2(\overline{AO} - \overline{AH}) =$

$= 2\left(2 - \sqrt{4-x^2}\right) = 4 - 2\sqrt{4-x^2}$

Quindi:

$\overline{AB} = 4$; $\overline{AD} = \overline{BC} = 2$; $\overline{DC} = 4 - 2\sqrt{4-x^2}$

Perimetro $= \overline{AB} + 2 \cdot \overline{AD} + \overline{DC} =$

$= 12 - 2\sqrt{4-x^2}$

$\Rightarrow y = 12 - 2\sqrt{4-x^2}$

GRAFICO

$y = 12 - 2\sqrt{4-x^2}$

con $4 - x^2 \geq 0 \Rightarrow -2 \leq x \leq 2$

$x =$ raggio $\Rightarrow x \geq 0 \Rightarrow 0 \leq x \leq 2$

$y = 12 - 2\sqrt{4-x^2} \leq 12$

Quindi:

$$\begin{cases} y = 12 - 2\sqrt{4-x^2} \\ 0 \le x \le 2 \\ y \le 12 \end{cases}$$

$$y - 12 = -2\sqrt{4-x^2} \Rightarrow (y-12)^2 = 4(4-x^2)$$

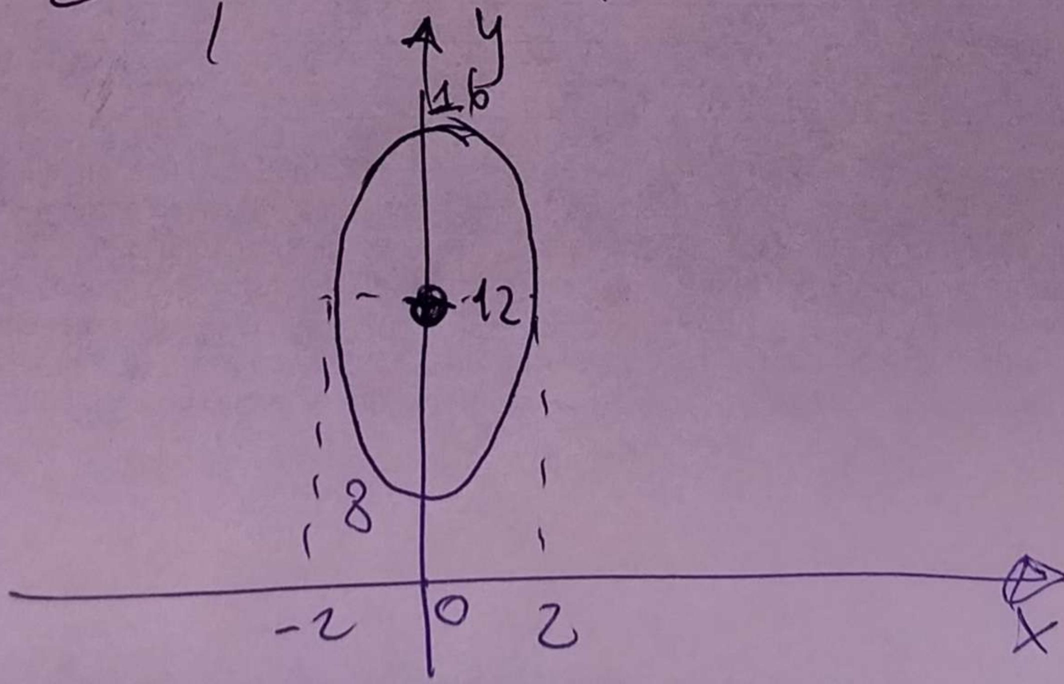
$$\Rightarrow \frac{(y-12)^2}{4} = 4 - x^2 \Rightarrow$$

$$\Rightarrow x^2 + \frac{(y-12)^2}{4} = 4$$

$$\Rightarrow \frac{x^2}{4} + \frac{(y-12)^2}{16} = 1$$

Ellisse avente il centro in $(0;12)$ e semiassi:

$a = 2$, $b = 4$



Relativamente al problema:

$$\begin{cases} 0 \leq x \leq 2 \\ y \leq 12 \\ y = 12 - 2\sqrt{4 - x^2} \end{cases}$$